

Verifying Identities

Pythagorean Identities

★ 1. $\sin^2 x + \cos^2 x = 1$

$-\sin^2 x = 1 - \cos^2 x$

$-\cos^2 x = 1 - \sin^2 x$

★ 2. $1 + \tan^2 x = \sec^2 x$

$-\tan^2 x = \sec^2 x - 1$

$-1 = \sec^2 x - \tan^2 x$

★ 3. $1 + \cot^2 x = \csc^2 x$

$-\cot^2 x = \csc^2 x - 1$

$-1 = \csc^2 x - \cot^2 x$

1. $\cot x = \cos x \csc x$

$$\cot x = \frac{\cos x}{1} \cdot \frac{1}{\sin x}$$

$$\boxed{\cot x = \cot x}$$

2. $2\sec^2 x = \frac{1}{1-\sin x} + \frac{1}{1+\sin x}$

$$2\sec^2 x = \frac{1+\sin x}{(1-\sin x)(1+\sin x)} + \frac{1-\sin x}{(1-\sin x)(1+\sin x)}$$

$$2\sec^2 x = \frac{(1+\sin x) + (1-\sin x)}{(1-\sin x)(1+\sin x)} = \frac{(1+\sin x) + (1-\sin x)}{1 + \cancel{\sin x} - \cancel{\sin x} - \sin^2 x}$$

$$\frac{(1+\cancel{\sin x}) + (1-\cancel{\sin x})}{1 - \sin^2 x} \xrightarrow{\text{Identity}} \frac{2}{\cos^2 x} = \boxed{2\sec^2 x}$$

3. $\frac{\sec x - 1}{\sec x + 1} \cdot \frac{\cos x - 1}{\cos x + 1} = \sec x + 1$

$$\frac{(\cos x + 1)(\sec x - 1)(\sec x + 1)(\cos x - 1)}{(\sec x + 1)(\cos x + 1)}$$

SKIP

$$\sin^2 x + \cos^2 x = 1$$

4. $\sin^2 x + \sin x + \cos^2 x - 1$

$$\cancel{1} + \sin x - \cancel{1}$$

$$= \boxed{\sin x}$$

5. $\csc x \div \cot x + \tan x$

$$\frac{\cancel{\cos x} \cancel{\cos x}}{\cancel{\cos x} \sin x} + \frac{\sin x \cdot \cancel{\sin x}}{\cancel{\cos x} \cdot \sin x}$$

Identity!

$$\csc x \div \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

$$\frac{1}{\sin x} \div \frac{1}{\sin x \cos x}$$

K C F

$$\frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x} \cos x}{1} = \boxed{\cos x}$$

6. $\csc x - \cot x \div \sec x - 1$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} \div \sec x - 1$$

Identity

$$\frac{1 - \cos x}{\sin x} \div \sec x - 1$$

$$\frac{\cancel{\sin x}}{\cancel{\sin x}} = 1 \div \frac{1}{\cos x} - \frac{1 \cdot \cos x}{1 \cdot \cos x}$$

Identity

$$1 \div \frac{1}{\cos x} - \frac{\cos x}{\cos x} = \frac{1 - \cos x}{\cos x} \rightarrow 1 \div \frac{\sin x}{\cos x} = \frac{1}{\tan x} = \boxed{\cot x}$$

7. $\cos x + \tan x \sin x$

$$\frac{\cos x}{1} + \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1} \rightarrow \frac{\overset{\cos x}{\cos x} \cdot \cos x}{\cos x} + \frac{\sin^2 x}{\cos x}$$

$$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \frac{\overset{\text{Identity}}{\sin^2 x + \cos^2 x}}{\cos x}$$

$$\frac{1}{\cos x} = \boxed{\sec x}$$