

Sum or Difference of Angles

Possible Formulas

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

1. Find $\sin(75^\circ)$

$$\begin{aligned}\sin(75^\circ) &= \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

2. Find $\cos(15^\circ)$.

$$15^\circ = 45^\circ - 30^\circ$$

$$\begin{aligned}\cos(15^\circ) &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

3. $\tan(75^\circ)$.

$$75^\circ = 45^\circ + 30^\circ$$

$$\begin{aligned}\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} &\rightarrow \frac{1 + \frac{1}{\sqrt{3}}}{1 - \left(1 \cdot \frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ 1 &= \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{3} + 1}{\sqrt{3}} \\ 1 &= \frac{\sqrt{3}}{3} - \frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{3} - 1}{\sqrt{3}} \\ &= \frac{(\sqrt{3} + 1)^2}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{3 + 2\sqrt{3} + 1}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}\end{aligned}$$

$$\frac{3 + 2\sqrt{3} + 1}{2} = \frac{4 + 2\sqrt{3}}{2} = \boxed{2 + \sqrt{3}}$$

4. $\sin(105^\circ)$

$$105 = 60^\circ + 45^\circ$$

$$\sin(105^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

5. $\cos(105^\circ)$

$$\cos(105^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

$$\tan \pm 80 = 0$$

6. $\tan(195^\circ)$

$$\tan(195^\circ) = \frac{\tan 180 + \tan 15}{1 - \tan 180 \tan 15}$$

$$\frac{0 + \tan 15^\circ}{1 - 0} =$$

$$\tan 15^\circ = \boxed{2 - \sqrt{3}}$$

$$\tan 45^\circ - \tan 30^\circ \rightarrow \text{Po Pit}$$

$$\frac{\frac{1}{1} - \frac{1}{\sqrt{3}}}{1 + \frac{1}{1} \cdot \frac{1}{\sqrt{3}}} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{3 - 1} = \frac{3 - 2\sqrt{3} + 1}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

7. $\sin(210^\circ)$ Q III Negative Sin

$$\sin(210^\circ) = \sin 180^\circ \cos 30^\circ + \cos 180^\circ \sin 30^\circ$$

$$(0)\left(\frac{\sqrt{3}}{2}\right) + (-1)\left(\frac{1}{2}\right)$$

$$0 + -\frac{1}{2} = \boxed{-\frac{1}{2}}$$

8. $\cos(300^\circ)$

$$360^\circ - 60^\circ$$

$$\cos(300^\circ) = \cos 360^\circ \cos 60^\circ + \sin 360^\circ \sin 60^\circ$$

$$(1)\left(\frac{1}{2}\right) + (0)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{\frac{1}{2}}$$

9. $\sin\left(\frac{\pi}{12}\right)$. $\frac{\pi}{12} \cdot \frac{180}{\pi} = \frac{180}{12} = 15^\circ$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

10. $\cos\left(\frac{5\pi}{12}\right)$.

$\frac{5\pi}{12} \cdot \frac{180}{\pi} = \frac{5 \cdot 180}{12} = 75^\circ$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

11. $\tan\left(\frac{\pi}{12}\right)$.

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12. $\sin\left(\frac{7\pi}{12}\right)$ $\frac{7\pi}{12} \cdot \frac{180}{\pi} = \frac{7 \cdot 180}{12} = 105^\circ$ QII

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

13. $\cos\left(\frac{7\pi}{12}\right)$ $\frac{7\pi}{12} = 105^\circ$ Q II, negative cos

~~cos 105~~

$$\cos\left(\frac{7\pi}{12}\right) = \cos\frac{\pi}{3} \cos\frac{\pi}{4} - \sin\frac{\pi}{3} \sin\frac{\pi}{4}$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

14. $\tan\left(\frac{11\pi}{12}\right)$

$\frac{11\pi}{12} = 165^\circ$ Q II, tan is negative

$\tan\frac{11\pi}{12} = \frac{\tan\frac{3\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{3\pi}{4} \tan\frac{\pi}{6}}$

$\frac{-1 + \frac{1}{\sqrt{3}}}{1 - (-1 \cdot \frac{1}{\sqrt{3}})} \rightarrow \frac{-1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}}$

$$\frac{-\sqrt{3} + 1}{(\sqrt{3} + 1) \cdot (\sqrt{3} - 1)}$$

$\frac{-(3 - \sqrt{3} - \sqrt{3} + 1)}{3 - 1}$

$\frac{-4 + 2\sqrt{3}}{2} = -2 + \sqrt{3}$